Exam Jan 2012 - Solution
Master Course: Tax Policy

1. Efficiency Cost

1.1. The instantaneous budget constraints are $c_1 + s_1 = w$, $(1+r(1-\tau))s_1 = c_2$. With a specific tax on second-period consumption instead of a tax on interest income, the second-period budget constraint is $(1+r)s_1 = (1+t)c_2$. Hence, both tax systems are equivalent if $\frac{1+t}{1+r} = \frac{1}{1+r(1-\tau)} \to t = \frac{\tau r}{1+r(1-\tau)}$.

1.2. Consider an increase in the tax rate t on good 1 to $t + \Delta t$. Recall the expression for EB:

$$EB(t) = [e(1+t, U) - e(1, U)] - th_2(1+t, U) \tag{1}$$

A second-order Taylor expansion of the term is

$$\begin{aligned} MEB &= EB(t + \Delta t) - EB(t) \quad (2) \\ &\simeq \frac{dEB}{dt}(\Delta t) + \frac{1}{2}(\Delta t)^2 \frac{d^2 EB}{dt^2} \quad (3) \end{aligned}$$

where

$$\begin{aligned} \frac{dEB}{dt} &= h_2(1+t, U) - t\frac{dh_2}{dt} - h_2(1+t, U) \quad (4) \\ &= -t\frac{dh_2}{dt} \quad (5) \\ \frac{d^2 EB}{dt^2} &= -\frac{dh_2}{dt} - t\frac{d^2 h_2}{dt^2} \quad (6) \end{aligned}$$

Thus, the approximated marginal excess burden is

$$\Rightarrow MEB = -t\Delta t \frac{dh_2}{dt} - \frac{1}{2}\left(\frac{dh_2}{dt} + t\frac{d^2 h_2}{dt^2}\right)(\Delta t)^2 \tag{7}$$

1.3. Compute the compensated demand function for second-period consumption h_2! The Lagrangian is $c_1 + (1+t)/(1+r)c_2 - \lambda(c_1 + \ln c_2 - \overline{u})$. The focs are (c_1) $1 - \lambda = 0$, (c_2) $(1+t)/(1+r) - \lambda/c_2 = 0$ and (λ) $c_1 + \ln c_2 = \overline{u}$. The first foc yields: $\lambda = 1$. Inserting the term into the second foc gives $h_2 = (1+r)/(1+t)$.

The first- and second-order derivative of h_2 w.r.t. t are $dh_2/dt = -(1+r)/(1+t)^2 < 0$ and $d^2h_2/dt^2 = 2(1+r)/(1+t)^3 > 0$. The compensated demand function for second-period consumption is convex in the tax rate. Ignoring the curvature property (i.e. setting $d^2h_2/dt^2 = 0$) in the computation overestimates the drop in compensated demand and thereby the marginal excess burden.

2. Optimal Taxation

2.1. The Lagrangian of the household's maximization problem is

$$\mathcal{L} = U(c_1, .., c_N, l) + \alpha(wl - (p_1c_1 + .. + p_Nc_N)),$$

where α is the Lagrange multiplier associated with the household budget constraint. The first order conditions are

$$U_{c_i} = \alpha p_i \text{ and } -U_l = \alpha w.$$

2,2. The household's optimization problem yields demand functions $c_1^*(p), ..., c_N^*(p)$, the labor supply function $l^*(p)$ and the indirect utility function $V(p)$ where $p = (w, p_1, .., p_N)$. Let's consider the effect of a higher tax (let's choose the tax on good i) on household utility $V(p)$. By the envelope theorem (Roy's identity) we have

$$\frac{\partial V}{\partial p_i} = -\alpha c_i^*.$$

Note, $\partial V/\partial p_i = \partial V/\partial t_i$ since $p_i = 1 + t_i$.

2.3. The government solves the maximization problem

$$\max V(p)$$

subject to the revenue requirement

$$\sum_{i=1}^{N} t_i c_i^*(p) = R.$$

The Lagrangian for the government is:

$$\mathcal{L}_G = V(p) + \lambda[\sum_i t_i c_i^*(p) - R],$$

where λ is the Lagrange multiplier associated with the government budget constraint.

Note, $\partial p_i/\partial t_i = 1$ since $p_i = 1 + t_i$. Thus, differentiating with respect to t_i is equivalent to differentiating with respect to p_i. As a matter of choice, differentiating with respect to p_i yields

$$\frac{\partial \mathcal{L}_G}{\partial p_i} = \frac{\partial V}{\partial p_i} + \lambda \left[c_i^* + \sum_{j=1}^{N} t_j \frac{\partial c_j^*}{\partial p_i} \right] = 0.$$

The first-order condition aligns the change in household utility due to a higher tax rate, $\partial V/\partial p_i$, to the rise in revenues expressed in utility units, $-\lambda(c_i^* + \sum_{j=1}^{N} t_j \partial c_j^*/\partial p_i)$.

2.4. When income effects are absent, compensated and uncompensated demand functions coincide, i.e. $c_j^* = h_j$. Further, with zero cross-price effects, the first-order condition reduces to (while making use of Roy's identity)

$$\frac{\partial \mathcal{L}_G}{\partial p_i} = -\alpha h_i + \lambda \left[h_i + t_i \frac{\partial h_i}{\partial p_i} \right] = 0.$$

The first-order condition can be reformulated as

$$\frac{t_i}{1+t_i} = -\frac{\lambda - \alpha}{\lambda}\frac{1}{\epsilon_i} \text{ where } \quad \epsilon_i = \frac{\partial h_i}{\partial p_i}\frac{1+t_i}{h_i}$$

Since $\alpha > 0, \lambda < 0$ and $\epsilon_i < 0$, the tax rate on any commodity is non-negative and varies inversely with its compensated demand elasticity. Commodities that are more elastic in demand are taxed at lower rates.

3. Optimal Capital Taxes

3.1. Government tax revenues in period t are

$$R_t = \tau^k r_t K_t + \tau^l w_t L_t.$$

3.2. Expanding the revenue term

$$R_t = \tau^k r_t K_t + \tau^l w_t L_t.$$

yields

$$R_t = (1 - 1 + \tau^k) r_t K_t + (1 - 1 + \tau^l) w_t L_t.$$

Using the definition of $\bar{r}_t$ and $\bar{w}_t$ (as given in the exercise), we have

$$R_t = (r_t - \bar{r}_t) K_t + (w_t - \bar{w}_t) L_t.$$

3.3. The Lagrangian of the government decision problem is

$$\mathcal{L} = V_1 + \sum_{t=1}^{\infty} \lambda_t (F_t(K_t, L_t) + K_t - C_t - G_t - K_{t+1}) + \sum_{t=1}^{\infty} \mu_t (b_{t+1} - (1+\bar{r}_t) b_t - \bar{r}_t K_t - \bar{w}_t L_t + F_t(K_t, L_t) - G_t).$$

λ_t and μ_t are the Lagrange multipliers associated with the resource constraint and public budget constraint in period t.

3.4. The first-order condition for K_{t+1} is

$$-\lambda_t + \lambda_{t+1}\left(\frac{\partial F_{t+1}(K_{t+1}, L_{t+1})}{\partial K_{t+1}} + 1\right) + \mu_{t+1}\left(-\bar{r}_t + \frac{\partial F_{t+1}(K_{t+1}, L_{t+1})}{\partial K_{t+1}}\right) = 0.$$

Note, with competitive markets firms use capital until

$$\frac{\partial F_{t+1}(K_{t+1}, L_{t+1})}{\partial K_{t+1}} = r_{t+1}.$$

Thus, the first-order condition is

$$-\lambda_t + \lambda_{t+1}(r_{t+1} + 1) + \mu_{t+1}(-\bar{r}_t + r_{t+1}) = 0.$$

3.5. Along a stationary trajectory, the interest rates (before and after tax) stay constant. Along a stationary trajectory, the discounted Lagrange multipliers λ_{t+1} and μ_{t+1} are related to their values in each period t by

$$\tilde{\lambda} = (1+\bar{r})^{t+1}\lambda_{t+1} \text{ and } \tilde{\mu} = (1+\bar{r})^{t+1}\mu_{t+1}.$$

Thus, the first-order condition reduces to

$$-\tilde{\lambda}(1+\bar{r})^{-t} + \tilde{\lambda}(1+\bar{r})^{-(t+1)}(r+1) + \tilde{\mu}(1+\bar{r})^{-(t+1)}(-\bar{r}+r) = 0.$$

3.6. Rearranging the latter equation, we get

$$(\tilde{\lambda}+\tilde{\mu})(r-\bar{r}) = 0$$

Since the Lagrange multipliers $\tilde{\lambda}$ and $\tilde{\mu}$ are positive, we have $r = \bar{r}$. This implies a zero tax on capital, $\tau^k = 0$.

Intuition:

- Deadweight loss of taxation rises with square of tax rate.
- With non-zero capital tax, we have an infinite price distortion between C_0 and C_t as $t \to \infty$
- Undesirable to have such large distortions on some decision margin.

4. Optimal Income Taxation

4.1. Consider a small tax reform, i.e. increase T' by $d\tau$ in a small income band $(z, z+dz)$. Three effects can be distinguished:

- Mechanical revenue effect

$$dM = dzd\tau(1-H(z))$$

- Mechanical welfare effect: Money-metric loss is dM by envelope theorem

$$dW = -dzd\tau(1-H(z))G(z)$$

- Behavioral effect: there is a substitution effect $\delta z < 0$ inside the small income band $[z, z+dz]$. This has the following impact on tax revenues:

$$dB = h(z)dz \cdot T' \cdot \delta z = -h(z)dz \cdot T' \cdot d\tau \cdot \varepsilon(z) \cdot z/(1-T')$$

  The last equality is due to rearranging the terms, observing that $dT' = d\tau$, and using the elasticity notation $\varepsilon(z) = \frac{dz}{d(1-T')}\frac{1-T'}{z}$.

- At the optimum $dM + dW + dB = 0$. Hence, the optimal tax schedule satisfies:

$$\frac{T'(z)}{1 - T'(z)} = \frac{1}{\varepsilon(z)} \left(\frac{1 - H(z)}{zh(z)} \right) [1 - G(z)]$$

4.2. Economic intuition for the three terms:

- Mechanical revenue effect: The effect shows how tax revenues rise simply because the tax rate increases, for a given behavior of households.
- Mechanical welfare effect: The effect is the mirror effect of the mechanical revenue effect since tax payers' utility decreases due to the higher tax payment.
- Behavioral effect: Tax payers adjust their behavior. They substitute away from supplying labor and consume more leasure. This erodes the income tax base and thus lowers tax revenues.

General properties of the optimal marginal tax rate $T'(z)$:

- $T'(z)$ is decreasing in $g(z')$ for $z' > z$ - the higher the welfare weight high-income housholds get, the lower is their marginal tax rate [redistributive tastes]

- $T'(z)$ is decreasing in $\varepsilon(z)$ - the more elastic the supply of income, the lower is the marginal tax rate [efficiency]

- $T'(z)$ is decreasing in $h(z)/(1 - H(z))$ - the higher the number of households that engage in tax avoidance (tax-induced substitution effect) relative to the number of households from which higher taxes can be collected, the lower is the marginal tax rate [density]

4.3. An EITC implies that labor supply is subsidized, i.e. $T' < 0$ over some income interval.

- Suppose $T' < 0$ in income band $[z, z + dz]$

- Increase T' by $d\tau > 0$ in income band $[z, z + dz]$

  - $dM + dW > 0$ because $G(z) < 1$ for any $z > 0$ (with declining $g(z)$ and $G(0) = 1$)

  - $dB > 0$ because $T'(z) < 0$ [smaller efficiency cost]

- Therefore $T'(z) < 0$ cannot be optimal

  - Marginal subsidies also distort local incentives to work

  - Better to redistribute using lump sum grant